

# Color Octet Contribution in Exclusive P-Wave Charmonium Decay

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Recent advances in our understanding of the higher-wave quarkonia have generated much interests in quarkonium physics. However most are devoted to inclusive decays and productions. Experimental data of several two-body exclusive decay channels of P-wave charmonia such as  $\pi\pi$  and  $p\bar{p}$  are available and some have recently been re-measured by the BES collaboration. It is not clear from the outset that color octet is needed for these exclusive channels. Indeed only color singlet has been used in the past and reasonable agreement with data was found. Contrary to these old results, we provide theoretical arguments for the inclusion of color octet and perform explicit calculations to back this up.

## 1. Introduction

Quarkonia are special hadronic systems in that they have the mass as an intrinsic large scale. This permits factorization, the separation of the hard from the soft scale physics. Therefore once supplemented with non-perturbative matrix elements in inclusive processes or hadronic wavefunctions in exclusive processes, physical processes involving heavy quarkonia can be calculated perturbatively. In this talk, we would like to re-examine some exclusive charmonium decay channels. There are several reasons for doing this. First being non-relativistic, the heavy quark-antiquark system is simpler than most of the lighter hadrons and therefore is good for the study of color confinement in this special case. If sufficiently understood, it provides a testing ground for our understanding of wavefunctions of selected lighter hadrons. Then recently several decay channels of P-wave charmonia have been measured or re-measured at the Beijing Spectrometer by the BES collaboration so there are new data available. Third if one looks up the existing calculations, it is immediately evident that they are rather out-of-date. Not only the QCD parameters have changed, the hadronic wavefunctions used have also been updated and better understood. Last but not least there is also

the emergence of the color octet higher states in quarkonium, their significance was revealed in [1,2]. The simplest decays are the two-body channels and so we will look at these of the P-wave  $\chi_J$  system.

## 2. Two-body P-wave charmonium decay

A well-known scheme for calculating exclusive processes is the hard scattering approach (sHSA) of Brodsky and Lepage [3] which became a kind of standard so that is what the small “s” stands for in the acronym. This scheme relies on the presence of a large momentum transfer to validate factorization so that the probability amplitude  $\mathcal{M}$  of the process in question can be written as a convolution of the hadronic distribution amplitudes  $\phi_h$  of the hadrons involved and a hard perturbative part  $T_H$  which serves as the conduit of the large momentum flow in the Feynman diagram between the initial and final hadrons. For a two-body decay process resulting in hadron  $h_1$  and  $h_2$ , this can be summarized in the following form

$$\mathcal{M} \sim f_{\chi_J} \phi_{\chi_J} \otimes T_H \otimes f_{h_1} \phi_{h_1} \otimes f_{h_2} \phi_{h_2} . \quad (1)$$

The  $f_h$ 's are the decay constants of the respective hadrons. The convolution here is over the light-cone momentum fractions. The partial decay width in terms of the above amplitude and the heavy charmonium mass  $M$  is essentially

$$\Gamma_{\chi_J \rightarrow h_1 h_2} \sim \frac{1}{M} |\mathcal{M}|^2 . \quad (2)$$

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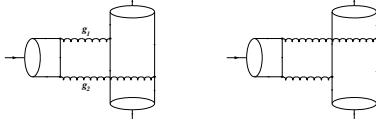


Figure 1. The graphs for calculating the color singlet contribution for the  $\chi_J$  decay into  $\pi\pi$ .

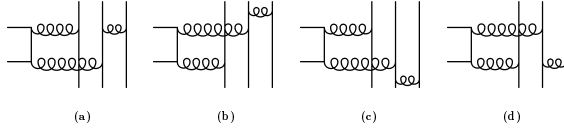


Figure 2. The color singlet graphs for the decay into  $p\bar{p}$ .

For definiteness we will examine the  $\chi_J$  decay into  $\pi\pi$  and  $p\bar{p}$ . Both of these have been measured and data are available from both Particle Data Group [4] and BES [5].

The calculations of the decay into the above two channels are quite reasonable with only two diagrams in the former and four in the latter process and so are not too complicated [6,7]. These are shown in Fig. 1 and Fig. 2. Using the sHSA and some rather well-known quantities such as the  $f_\pi$ ,  $\chi_J$  radial wavefunction  $R'_P(0)$ ,  $m_c$  etc. the partial widths can be obtained. One gets the results shown in Table 1. Comparing the partial widths  $\Gamma^{(1)}$  coming from the  $c\bar{c}$  state to those from experiments, one can see that the former are well below the data.

In order to make sure that these small values are not the result of the imperfection of the scheme used. They can be calculated again using an improved scheme (mHSA) of Sterman et al [8,9]. This scheme comes from modifying the sHSA by introducing transverse momentum effects, radiative correction in the form of Sudakov factor  $S$  and utilizing whole hadronic wavefunctions  $\psi_h$ . The probability amplitude can now be

Table 1

Partial decay widths obtained within the sHSA scheme.

$J$	$\Gamma_{\chi_J \rightarrow \pi\pi}$ [keV]		
	$\Gamma^{(1)}$	PDG	BES
0	15.30	$105.0 \pm 30$	$67.06 \pm 17.3$
2	0.841	$3.8 \pm 2.0$	$3.04 \pm 0.73$

$J$	$\Gamma_{\chi_J \rightarrow p\bar{p}}$ [eV]		
	$\Gamma^{(1)}$	PDG	BES
1	3.15	$75.68 \pm 10.5$	37.84
2	12.29	$200.00 \pm 20.0$	118.00

Table 2

Partial decay widths obtained within the mHSA scheme.

$J$	$\Gamma_{\chi_J \rightarrow \pi\pi}$ [keV]		
	$\Gamma^{(1)}$	PDG	BES
0	8.22	$105.0 \pm 30$	$67.06 \pm 17.3$
2	0.41	$3.8 \pm 2.0$	$3.04 \pm 0.73$

$J$	$\Gamma_{\chi_J \rightarrow p\bar{p}}$ [eV]		
	$\Gamma^{(1)}$	PDG	BES
1	2.53	$75.68 \pm 10.5$	37.84
2	16.58	$200.00 \pm 20.0$	118.00

summarized as

$$\mathcal{M} \sim \psi_{\chi_J} \otimes T_H(\alpha_s) \otimes \psi_{h_1} \otimes \psi_{h_2} \otimes \exp\{-S\}. \quad (3)$$

The convolution is now over not just light-cone fractions but also internal transverse momenta. Furthermore the coupling  $\alpha_s$  is part of the convolution whose scale is determined dynamically by the momentum flow, whereas in sHSA it is merely a constant. The importance of this was discussed in [7,10]. The results shown in Table 2 agree essentially with those in Table 1 on the fact that the theoretical results are too small. Of course, one could exploit the uncertainties in the various parameters to try to enlarge the calculated results but this proved to be fruitless. They remain below the data by factors of two or more [6,7,11,12].

### 3. What is the source of the problem?

The previous section showed that it was not possible to explain the experimental data from

the decay of the P-wave  $c\bar{c}$  system. In the search for the source of the problem, one cannot help but wonder if color octet is the answer. After all color octet seems these days to be inextricably intertwined with quarkonium physics in inclusive processes ever since it was first introduced in [1,2] to cancel the infrared divergence found in the inclusive  $\chi_J$  decay [13]. Strangely in the world of exclusive quarkonium processes, color octet does not seem to exist at all. In fact it has been totally neglected. A simple argument easily reveals that color octet must also be important in exclusive decays, albeit it is not sufficient to claim that it is so in the two-body channels. The argument relies on the fact that the sum of the partial widths of all channels must be equal to the total inclusive hadronic width which can be more precisely expressed in the form

$$\sum_{i \in \text{channels}} \Gamma_i = \Gamma_{\text{incl}}. \quad (4)$$

If none of the  $\Gamma_i$  requires the color octet contribution, then there would be a contradiction because it is known to be of importance and needed on the right-hand-side (r.h.s). However the contradiction can be avoided provided color octet is needed in at least one channel, it need not be the two-body channels that we are interested in. So *a priori* for our problem at hand, it is not clear that color octet is the answer. Furthermore there is the usual folklore that says higher state contributions are suppressed in large momentum transfer processes which seems to oppose the inclusion of the octet contribution.

#### 4. The large mass dependence of the decay amplitudes

The pros and cons of color octet as the answer to our problem discussed in the last section makes the issue very unclear. To come to a resolution, it is necessary to look closely at the charmonium system. The hadronic decay is via annihilation into gluons. The  $c\bar{c}$  pair has to come close together for this at a small distance  $l \sim 1/M$  because of the large charmonium mass  $M$ . For a P-wave charmonium, the  $L = 1$  orbital angular momentum tends to force the pair apart there-

fore it is harder for the pair to annihilate. This is manifested in the vanishing of the P-wave wavefunction at the origin  $\psi_P(0) = 0$ . Comparing a S-wave to a P-wave system, assuming all else being equal, the only difference in magnitude of the decay amplitude lies entirely with the wavefunctions at small distance. Because the P-wave wavefunction vanishes there, its derivative enters the decay amplitude in its place. The following shows the relevant quantities that enter the amplitudes and the same after transforming into momentum space.

$$\begin{aligned} S : \quad & \psi_S(l \sim 0) \quad \rightarrow \quad \tilde{\psi}_S(k) \\ P : \quad & \psi_P(l \sim 0) \simeq l\psi'_P(l \sim 0) \quad \rightarrow \quad \frac{k}{M}\tilde{\psi}_P(k) \end{aligned}$$

This shows the P-wave quantity carried with it a power of  $1/M$  so a P-wave charmonium is suppressed at the valence level already by  $M$  in comparison to a S-wave. In view of this power suppression, it is perhaps useful to examine and compare the  $M$  dependence of the decay amplitudes.

In the calculations of exclusive process, one has to deal all too frequently with decay constants of the hadrons involved. So it should be more relevant to look at the decay amplitudes in terms of  $f_h$ 's and  $M$ . The sHSA scheme is therefore more convenient for this purpose. Since the strongest dependence on  $M$  is powerlike, it is useful to look at dimensions. From Eq. (2)  $\mathcal{M}$  is of mass dimension one so we have the following equation of dimensions

$$[\mathcal{M}] = [\text{mass}]^1 = [f_{\chi_J}][f_{h_1}][f_{h_2}][T_H]. \quad (5)$$

The r.h.s. shows all the quantities that carried a mass dimension. The hard part  $T_H$  has some hidden power dependence of the form  $M^{-p}$  for some number  $p$ . Together with the  $f_h$ 's, they make up for the dimension of  $\mathcal{M}$ . The  $M$  dependence of  $\mathcal{M}$  will be known, once the mass dimensions of the  $f_h$ 's are determined. Using the relation of  $f_h$  to the associated wavefunction  $\psi_h$

$$\begin{aligned} & f_h \phi_h(x) \\ &= \int^Q \prod_i^{N-1} \frac{d^2 \mathbf{k}_{\perp i}}{(2\pi)^2} \psi_h(x; \mathbf{k}_{\perp 1}, \dots, \mathbf{k}_{\perp N-1}) \quad (6) \end{aligned}$$

and the normalization of  $\psi_h$

$$\int \prod_i^{N-1} dx_i \frac{d^2 \mathbf{k}_{\perp i}}{(2\pi)^2} |\psi_h(x; \mathbf{k}_{\perp 1}, \dots, \mathbf{k}_{\perp N-1})|^2 = \text{prob.}, \quad (7)$$

one can deduce the dimension of  $f_h$ . It can be summarized by

$$[\psi_h] = [\text{mass}]^{1-N}, \quad [f_h] = [\text{mass}]^{N-1+L} \quad (8)$$

where  $N$  is the number of constituents in the hadron  $h$  described by  $\psi_h$  and  $L$  is the orbital angular momentum of  $\psi_h$ . The decay constants for our hadrons are then

$$[f_\pi] = [\text{mass}]^1, \quad [f_p] = [f_{\chi_J}^{(1)}] = [f_{\chi_J}^{(8)}] = [\text{mass}]^2.$$

For the  $\chi_J$  decays, the color singlet and octet amplitude therefore have the following  $M$  dependence

$$\begin{aligned} \mathcal{M}_{\chi_J \rightarrow \pi\pi}^{(1)} &\sim M \frac{f_{\chi_J}^{(1)}}{M^2} \left(\frac{f_\pi}{M}\right)^2 \sim \frac{1}{M^3} \\ \mathcal{M}_{\chi_J \rightarrow \pi\pi}^{(8)} &\sim M \frac{f_{\chi_J}^{(8)}}{M^2} \left(\frac{f_\pi}{M}\right)^2 \sim \frac{1}{M^3} \\ \mathcal{M}_{\chi_J \rightarrow p\bar{p}}^{(1)} &\sim M \frac{f_{\chi_J}^{(1)}}{M^2} \left(\frac{f_p}{M^2}\right)^2 \sim \frac{1}{M^5} \\ \mathcal{M}_{\chi_J \rightarrow p\bar{p}}^{(8)} &\sim M \frac{f_{\chi_J}^{(8)}}{M^2} \left(\frac{f_p}{M^2}\right)^2 \sim \frac{1}{M^5}. \end{aligned}$$

For both decay channels, singlet and octet contribution have the same power dependence in  $M$ . There is no large  $M$  suppression of the octet in comparison to the singlet. The usual suppression of the higher state is nullified by the suppression of the P-wave valence state due to angular momentum.

To elucidate this further, we can turn to the same decay channels of the S-wave  $J/\psi$ . Now the  $J/\psi$  decay constants have the dimensions

$$[f_{J/\psi}^{(1)}] = [\text{mass}]^1, \quad [f_{J/\psi}^{(8)}] = [\text{mass}]^3.$$

The octet constant has a dimension of three instead of two is because the three-body wavefunction must have  $L = 1$  to make up the right quantum numbers for  $J/\psi$  as explained in [7]. The

$J/\psi$  decay amplitudes have the dependence

$$\begin{aligned} \mathcal{M}_{J/\psi \rightarrow \pi\pi}^{(1)} &\sim M \frac{f_{J/\psi}^{(1)}}{M} \left(\frac{f_\pi}{M}\right)^2 \sim \frac{1}{M^2} \\ \mathcal{M}_{J/\psi \rightarrow \pi\pi}^{(8)} &\sim M \frac{f_{J/\psi}^{(8)}}{M^3} \left(\frac{f_\pi}{M}\right)^2 \sim \frac{1}{M^4} \\ \mathcal{M}_{J/\psi \rightarrow p\bar{p}}^{(1)} &\sim M \frac{f_{J/\psi}^{(1)}}{M} \left(\frac{f_p}{M^2}\right)^2 \sim \frac{1}{M^4} \\ \mathcal{M}_{J/\psi \rightarrow p\bar{p}}^{(8)} &\sim M \frac{f_{J/\psi}^{(8)}}{M^3} \left(\frac{f_p}{M^2}\right)^2 \sim \frac{1}{M^6}. \end{aligned}$$

So the octet contribution is suppressed by  $M^{-2}$  in both cases in the amplitude. One can therefore legitimately neglect the color octet contributions in  $J/\psi$  but not in  $\chi_J$ .

## 5. Color octet contributions

In this section, we briefly outline and discuss the calculation of the octet contributions. The calculation can be done in either sHSA or mHSA scheme in principle, in practice it is easier to use sHSA than mHSA but the latter has important advantages discussed in [7,10,12,14]. To some degree the choice of scheme depends also on the final hadrons. The simpler  $\pi\pi$  is manageable in mHSA but not the  $p\bar{p}$  channel which is too complicated. It is better to work in sHSA for calculating the latter.

Since the color octet wavefunctions are not known, they have to be constructed. To do this the associated decay constants  $f_{\chi_J}^{(8)}$  must be determined by fitting and the light-cone fractions' distribution fixed in a sensible manner. Numerically the latter has to be checked to see if it is sensible. Then there is the presence of the constituent gluon from the octet state of the charmonium which has to be considered. The usual valence color singlet  $c\bar{c}$  state is straight forward to deal with since they cannot appear in the final state and easily eliminated via annihilation. The constituent gluon on the other hand can be a part of the final hadron by becoming a constituent of a higher state or it can end in the hard perturbative part  $T_H$ . It was eventually decided that the dominant contribution would be for it to end on  $T_H$ .

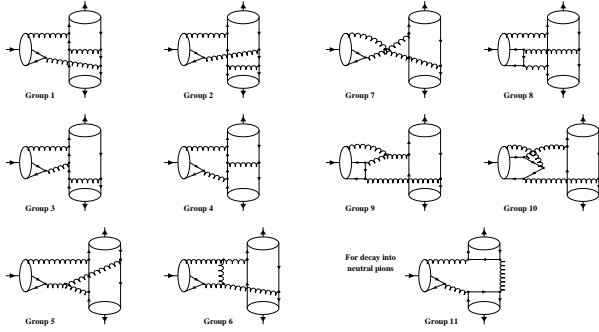


Figure 3. The groups of graphs that contribute to the decay into  $\pi\pi$ .

The contribution involving the higher state of a final hadron would be suppressed both by large momentum flow within HSA and the presence of the additional higher state. To complete the calculation of  $T_H$ , all possible feynman graphs must be found. Unlike the color singlet contribution, there are now a lot more diagrams. Because of C-parity, the color singlet state must annihilate into two or more gluons. The  $c\bar{c}$  in the color octet on the other hand is protected from the C-parity constraint by the presence of the constituent gluon and can therefore annihilate through one gluon. This leads to quite a number of diagrams. Additionally by allowing the constituent gluon to end in the hard part, it must be attached to all possible allowed positions of  $T_H$ . These all contribute to the number of graphs. In the end, they can be organized into groups and calculated by computer. Some examples are shown in Fig. 3 and Fig. 4. The first figure here is from [6]. The constituent gluon is yet to be included in the latter. The graphs shown in this one form the basis from which the groups are to be generated. It remains to convolute the  $\phi_h$ 's in the sHSA or  $\psi_h$  in the mHSA and  $T_H$  together to get the partial widths for the different two-body channels. The details for the calculation of the  $\chi_J$  decay into  $\pi\pi$  are to be found in [6,11] and those for  $\chi_J$  decay into  $p\bar{p}$  are in [7,12].

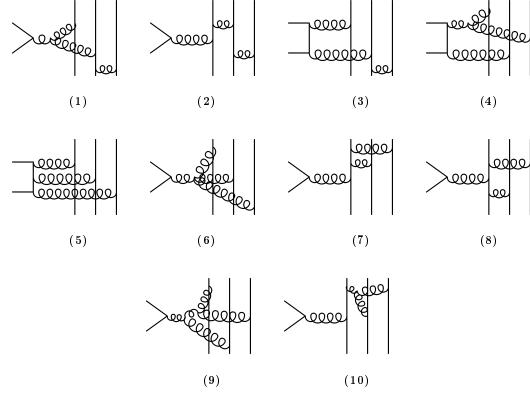


Figure 4. The groups that contribute to the decay into  $p\bar{p}$ . The constituent gluon is not drawn here.

## 6. Combining color singlet and octet contributions

The arguments in Sec. 4 show that for P-wave charmonium, color octet and singlet are of equal weight and must both be included. With the calculation of the singlet contribution in Sec. 2 and the octet contribution in Sec. 5, the two can now be combined to give, from a theoretical point of view, the proper partial widths of  $\chi_J$ . The results are tabulated in Table 3<sup>2</sup>. Notice that the experimental measurements are different between the PDG and BES data. It is up to the experimentalists to reconcile this in the future. The important point here is that by including the octet contributions, the theoretical widths are in much better shape for explaining the measured data and also in accordance with theoretical expectation. With hindsight it is not entirely surprising that color octet is required in the two-body decay channels since in inclusive decays it is needed already at the leading order for  $\chi_1$  and next-to-leading order for  $\chi_0$  and  $\chi_2$ .

<sup>2</sup>These results for the decay into proton-antiproton are final. They supersede all previously reported preliminary values in [10,14].

Table 3  
Singlet and singlet-octet combined partial widths.

$J$	$\Gamma_{\chi_J \rightarrow \pi\pi}$ [keV]			
	$\Gamma^{(1)}$	$\Gamma^{(1+8)}$	PDG	BES
0	8.22	45.4	$105.0 \pm 30$	$64.0 \pm 21.0$
2	0.41	3.64	$3.8 \pm 2.0$	$3.04 \pm 0.73$

$J$	$\Gamma_{\chi_J \rightarrow p\bar{p}}$ [eV]			
	$\Gamma^{(1)}$	$\Gamma^{(1+8)}$	PDG	BES
1	3.15	56.27	$75.68 \pm 10.5$	37.84
2	12.29	154.19	$200.00 \pm 20.0$	118.00

## 7. Summary

In this talk, we showed with new calculations that valence color singlet state of P-wave charmonia alone is insufficient to account for experimental measurements of the partial decay widths in contradiction to those done in the past predating the emergence of the color octet state in quarkonium physics. Normally higher states should be suppressed but this was shown to be circumvented because of angular momentum suppression of the valence state with  $L = 1$  which brought it down to the same level of the octet state. Power dependence in the heavy charmonium mass arguments revealed that it was inconsistent to perform P-wave charmonium calculations without including the color octet. This was eventually supported by explicit calculations. The anomaly of the situation in charmonium physics between inclusive and exclusive processes pointed out was thus reconciled. The arguments given here can be straight forwardly generalized to even higher-wave quarkonia. Calculations involving the latter require the inclusion of not only

the next higher states but also those above them. Therefore one has to be very careful when dealing with quarkonia whose valence states have non-zero orbital angular momentum.

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